AN APPROXIMATE GOAL CONSTRAINT MODEL FOR AMMUNITION INVENTORY

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THESIS

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ABSTRACT

The paper contends that the standard economic approach to inventory control may not be as valid in a military context as a goal constrained model which optimizes a performance criteria. The argument is applied to an ammunition system in a combat zone and a model is formulated to minimize expected on-hand inventory subject to constraints on required protection and order size or frequency. The model assumes that orders do not cross. Two examples, using Marine Corps data, were tested by a computer simulation that permitted orders to cross. The simulation showed that predicted average on hand levels were accurate but predicted protection was conservative. It was concluded that the model is best solved by simulation in view of the uncertainties caused by stochastic lead times and the skewness of lead time demand.

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I. INTRODUCTION

A. BACKGROUND

The usual approach to inventory control problems is to formulate and evaluate a model in economic terms. That is, the model is constructed to maximize an objective function that is expressed in units of dollars and cents. The standard formulation maximizes profit, which, of course, in the inventory situation means minimizing costs since it is generally assumed that the revenue portion of the profit picture is independent of the inventory activity. In any event, after assuming that the pertinent costs are known or can be quantified, the economic models derive a cost expression which is then minimized with respect to the decision variables. The values of the decision variables which result in the minimum cost of operating the inventory system are the optimum solution to the problem being studied.

However, these economic models are not entirely satisfying when applied to a military inventory context. There are three basic objections. First, military inventory managers do not have the flexibility in shifting funds or personnel that their civilian counterparts have. That is, costs which may be variable in a civilian system may be fixed, either legally or practically, in the military by budgets, Tables of Organization and Equipment and manning level considerations. Thus, the major factors of opportunity costs, personnel costs and equipment costs are not variable in response to changes in inventory policy except in the very long run. Next, since military organizations do not normally conduct the necessary accounting, it is a formidable job to accurately assess the costs necessary to solve cost-minimizing models. A model that does not require cost estimates

inherits an enormous benefit on this basis alone. Finally, it is not at all obvious that the objective of the military manager is to minimize the variable costs of an inventory system even though it is argued that minimizing costs is equivalent to increased efficiency and service.

It is the conjecture of this paper that there is an alternative way of stating the objectives of an inventory system in a manner more consistent with daily operations and with criteria upon which the system and the manager's performance are judged. This alternative may be described as a goal-constrained model as opposed to a cost-minimizing model. Such a model is formulated to optimize some performance criteria for the system subject to certain constraints. These constraints are of two kinds; goals imposed upon the system by some form of directive and physical resource limitations.

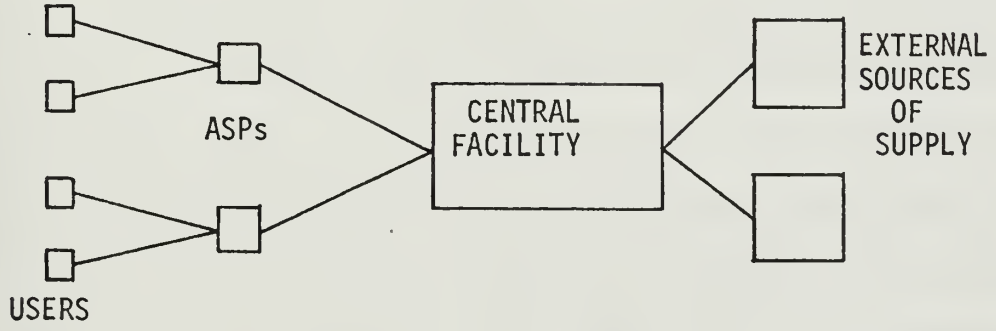
B. SYSTEM DESCRIPTION

Goal constraint models can be formulated in many different ways depending upon the choice of goals and the level of the system being studied. Clearly a model of an aggregate service-wide ammunition distribution system has to concern itself with procurement/production and budgetary constraints. A goal formulation of such a system was given by Tully [6].

In short, one must consider the precise nature of the system being analyzed, the characteristics of its operations and the system goals, either stated formally or surmised by observing day to day operations, in order to formulate an objective and state the important constraints. It is therefore necessary at this juncture to describe the system to be analyzed in this paper.

The system consists of the operation of a single central ammunition inventory facility serving all units in a combat theatre of operations. Note that "single" does not necessarily imply one geographic location. What is implied is that this facility receives all class V(W) material (ground ordnance, as opposed to aviation ordnance) delivered to the theatre of operations from external sources and, in turn, issues the ammunition to using units via subordinate Ammunition Supply Points, (ASPs), in the ammunition distribution system. Such a facility could easily be imagined merely as a paper or record keeping organization.

Figuratively, the facility looks like this;



The manager of this ammunition facility has the following responsibilities;

1. Requisition all class V(W) material for all subordinate units in the ammunition distribution system.

2. Issue ammunition to outlying Ammunition Supply Points in the theatre of operations according to directives of the responsible commander.

This paper is attempting to generalize a description of the ammunition management system in effect for III Marine Amphibious Force units in I Corps Area, RVN. Thus, ASP Danang is analogous to the facility being analyzed and the manager of the system we are discussing

would correspond to the Ammunition Officer, G-3, Force Logistics Command, III MAF. See Marine Corps Ammunition Management System, Vietnam, [Ref. 8], for a precise description of the situation in effect in the period, October 1968 to June 1969.

One important characteristic of the system is that it functions in a protracted combat environment. The model described later does not pertain to the classical amphibious operation of short duration. Rather, it pertains to conflicts where Marine forces are committed to long term operations since the performance of the system indicated by the model may not be realized in any given short period. It is only over the long haul that the system can be expected to perform as advertised.

Further, it is assumed that the ammunition distribution system has been stabilized. That is, on the demand side the organization of subordinate ASPs has been accomplished and the number of units being supported is not expected to change drastically. However, the requirement for stability is more important on the supply side. It is assumed that procurement systems are geared up to meet demand, the distribution system from production source to the ammunition dump is established and the rapid buildup triggered by the onset of hostilities is complete so that the system is relatively stable.

What is being said here in terms of inventory management is that demand is amenable to observation and, more importantly, forecasting, and that differences in lead times are due to variations inherent in the system's operations and not due to violent changes in the system itself.

One salient characteristic of such a system, evident from III MAF operations, is long lead times. That is, the time from the preparation

of a requisition until the ammunition is delivered to the dump is quite large compared to the interval between requisitions. Thus, one can expect several requisitions outstanding at any given time.

So, the system is a single central ammunition dump operating in a protracted combat environment facing large demands on one side and long lead times on the other. The manager of the facility has to order ammunition efficiently to meet the demands. To do this, the responsible officer has to decide when to order and how much to order.

There are two basic approaches to this problem. One is the continuous review policy, where the manager orders a fixed amount, Q, as soon as the inventory level drops to a reorder level, R. The alternative is the periodic review policy where the manager reviews his stocks after a fixed period, T, and orders the difference between his inventory position and an order-up-to level, R. The problem is to determine for either case the values of those decision parameters that will optimize the operation of the inventory system with respect to its objectives.

C. THE GOAL CONSTRAINT APPROACH

The approach presented here is raised as an alternative to the cost minimization model on the basis of these three factors.

1. Most of the costs experienced by the Marine Corps in running an ammunition distribution system can be described as either sunk or comparatively fixed and do not respond to changes in inventory decision rules.

2. A properly chosen goal constraint model will accomplish the same results as a cost-minimization model, e.g. minimize the average inventory and therefore the costs associated with holding inventory modified by a requirement not to be out of stock "too often."

3. A goal constraint model is more relevant to the operations of a military ammunition distribution system and, in particular, to the matters of concern to the responsible officer.

This approach is receiving attention in the Navy supply system and in military applications of inventory research because of its relevance to the military inventory problem. For example, see Refs. 6, 7, and 10. The method has a practical benefit because ammunition distribution goals and constraints lend themselves to a relatively simple mathematical formulation. As will be seen, the application of the model to reality presents some difficulties, but these same difficulties would pertain to the cost-minimization formulation as well.

The point was made that a goal constraint model can address itself to factors which are of direct concern to the officer responsible for the ammunition distribution system. Such an officer is not accountable to his superiors on a dollar basis nor does he frame his policies on that basis. Matters of concern to the responsible officer are operating characteristics such as the average amount of on-hand inventory, the frequency with which a certain item is out of stock, the frequency with which the available personnel can review the on-hand stock, the administrative workload capacity and the nature of his decision rules -- are they simple or self-defeatingly complex?

It seems intuitively satisfying that such an ammunition operation is based on the following or similar considerations. If left alone, the responsible personnel would like to reduce the on-hand inventory as much as possible. By so doing, the amount of handling required would be reduced with a resulting decrease in maintenance problems. In addition, proprietary responsibilities such as police, safety inspection and

security patrols would require less time. The ease of making a physical inventory would be enhanced resulting in improved record accuracy, system efficiency and response. Finally, the potential loss to enemy activity or disaster would be decreased. In fact, if the on-hand inventory could be reduced to an average of zero so that the operation was essentially on paper, the ammunition system manager's job would become almost pleasant.

However, the ammunition distribution system has an operational goal to meet that is specified by directives from higher authority and which is couched in terms of having sufficient ammunition on hand to meet potential future commitments to a specified degree. For example, the system may be required to keep so many days of ammunition on hand, staged at different echelons of the distribution system. The essence of this goal constraint is that higher authority has established a level of protection that the inventory system must meet.

Finally, there is some reason why the inventory manager cannot expect instantaneous shipment of any amount of ammunition needed. Instead, he faces an operating constraint on order size and frequency.

Thus, the responsible individual is trying to minimize his average on-hand inventory subject to meeting a specified level of protection against stockout and having to operate within resource constraints on the number of orders he can send and the size of the order he can realistically expect to receive.

II. THE MODEL

A. INTRODUCTION

Chapter I discussed an alternate approach for analyzing an ammunition inventory system in which the problem is to formulate a mathematical model which minimizes the average or expected on-hand inventory subject to meeting a protection goal and a resource constraint on the frequency and/or size of the orders. This chapter will formulate such a model. Following chapters will use empirical data taken from Ref. 8 to solve the model and discuss the implications of the approximations made.

It is necessary at this point to introduce the notation to be used for the remainder of the paper.

B. NOTATION

OH	On-hand inventory
BO	Number of rounds backordered at any given time
OO	Total amount on order at a given time
IP	Inventory position. (IP=OH+OO-BO)
Q	Fixed order quantity
R	Reorder trigger level or order up to level
S	Safety stock level
T	Review cycle length, or time between orders
x	Random variable representing demand per unit time. (e.g., rounds/day)
f(x,t)	Probability density function of x over a time interval (0,t)
τ	Random variable representing lead time
$g(\tau)$	Probability density function of lead time

$h(x,\tau+t)$ Expected value of $f(x,\tau+t)$ taken over the lead time distribution. (for t=0, this is the marginal distribution of lead time demand) i.e. $h(x,\tau+t) = \int_{-\infty}^{\infty} f(x,\tau+t)\, g(\tau)\, d\tau$.

U_x Mean demand per unit time

V_x Variance of demand per unit time

U_τ Mean lead time

V_τ Variance of lead time

U Mean lead time demand

V Variance of lead time demand

E(OH) Expected value of on-hand inventory

P(out) Probability of being out of stock at a random moment in time.

C. MODEL ASSUMPTIONS

In symbols, the problem can be stated:

Minimize $E(OH)$

Subject to: $P(out) \leq e$

$E(T) \geq T'$

where e is the acceptable level of the probability of being out of stock and T' is the minimum feasible number of days between orders.

The first assumption that must be made is that the various types of ammunition can be treated independently. The assumption implies that the different kinds of ammunition are not related with respect to meeting any constraint on the system's operation. In short, the solution to the multi-item inventory problem is obtained by solving the model for each item separately. This assumption is defensible since there is no overall constraint such as warehouse space or total investment. Note that the assumption does not imply that the different demand distributions are not correlated. Certain ammunitions no doubt are correlated but correlated demands do not affect the model since only the demand distribution

for each specific ammunition will be needed in the model. The effect of substitutability can be accounted for in the assignment of e. This assumption permits the capability of assigning different levels of protection to munitions based upon an assessment of the item's requirement for protection.

The situation being studied is one where the expected value of the lead time is greater than the length of the cycle between the placement of orders so that, in general, there is more than one order outstanding at any given time. This gives rise to the possibility of orders crossing en route to the dump. The empirical lead time distribution of Ref. 8 further reinforces this potential complication. Moreover, the case where orders can cross does not lend itself to analytical formulation in the present state of the theory according to Hadley [3] p. 202. Accordingly, it is assumed that orders do not cross.

Further necessary assumptions are:

1. Lead times are independent of demand and each other.

2. Demand per unit time is independent from one time period to the next. This is considered a valid assumption in a combat zone that is characterized by numerous small unit operations. The assumption of independent daily demands may not be as valid under the conditions of a coordinated theatre-wide offensive where day to day operations are likely to be influenced by previous days' activities.

So far, we have made one particularly critical assumption; orders do not cross. Thus, all orders arrive in sequence. A second critical part of the formulation is an assumption, made now, that the expected amount of backorders is small and can therefore be overlooked in the calculation of E(OH).

These elements combine to make the formulation an approximate one. Because we are dealing with decision rules that result from an approximation, (a characteristic of most pragmatic decision rules) we cannot be assured that the resulting solution provides the system with exact answers. To be objective and thorough, it is then necessary to test those decision rules. Therefore, the analytical results of the model were tested by a computer simulation in GPSS of the system's operations using selected data from Ref. 8 (high and low demand).

D. FORMULATION

The following argument has its genesis in Hadley [3]. Only the derivation of the periodic review case will be presented as this is the method currently in use. Nonetheless, since ammunition stock is reviewed daily as a matter of policy and the ammunition distribution system has recently been automated, it is now practicable for a continuous review policy to be put into effect. In theory, a continuous review policy performs better than a periodic review policy. The derivation of results is completely analogous for continuous review and the interested reader can go to the heuristic model of Chapter IV in Hadley [3] for the necessary approach. Accordingly, only final results for the continuous review case will be given.

1. Periodic Review

Reviews are made after the passing of a fixed period, T, and an amount is ordered sufficient to bring the amount on hand and on order up to R. The following discussion refers to the representation of the inventory cycle given in Figure II.1.

Consider the system at time t_0 just as an order is being placed. That order is received at time t_1 after time τ_1 has elapsed. By assumption,

everything on order at time t_0 has arrived at time $t_1 (t_1 = t_0 + \tau_1)$ and nothing not on order at t_0 has arrived by t_1. To maintain the safety stock at S, the demand in a lead time plus a review must equal R-S, or the expected value of S is equal to $R - (U_\tau + T)U_X$. Thus, if backorders are rare and can be ignored, the expected amount on hand is

$$S + \frac{TU_X}{2}$$

since the expected value of an order is $T\,U_X$ (the expected demand in T days).

Therefore:

$$E(OH) = R - U - \frac{TU_X}{2}\,.$$

In the periodic review case, however, one cannot expect the next order to arrive after the receipt of the order made at t_0 until $t_0 + U_\tau + T$. Accordingly, the probability of being out of stock just prior to that receipt is the probability that demand in a lead time plus one review period is greater than R, i.e.,

$$P(\text{out}) = \int_R^\infty \int_{-\infty}^\infty f(x,\tau_2+T)\, g(\tau_2)d\tau_2 dx = \int_R^\infty h(x,\tau+T)dx$$

Thus, we wish to

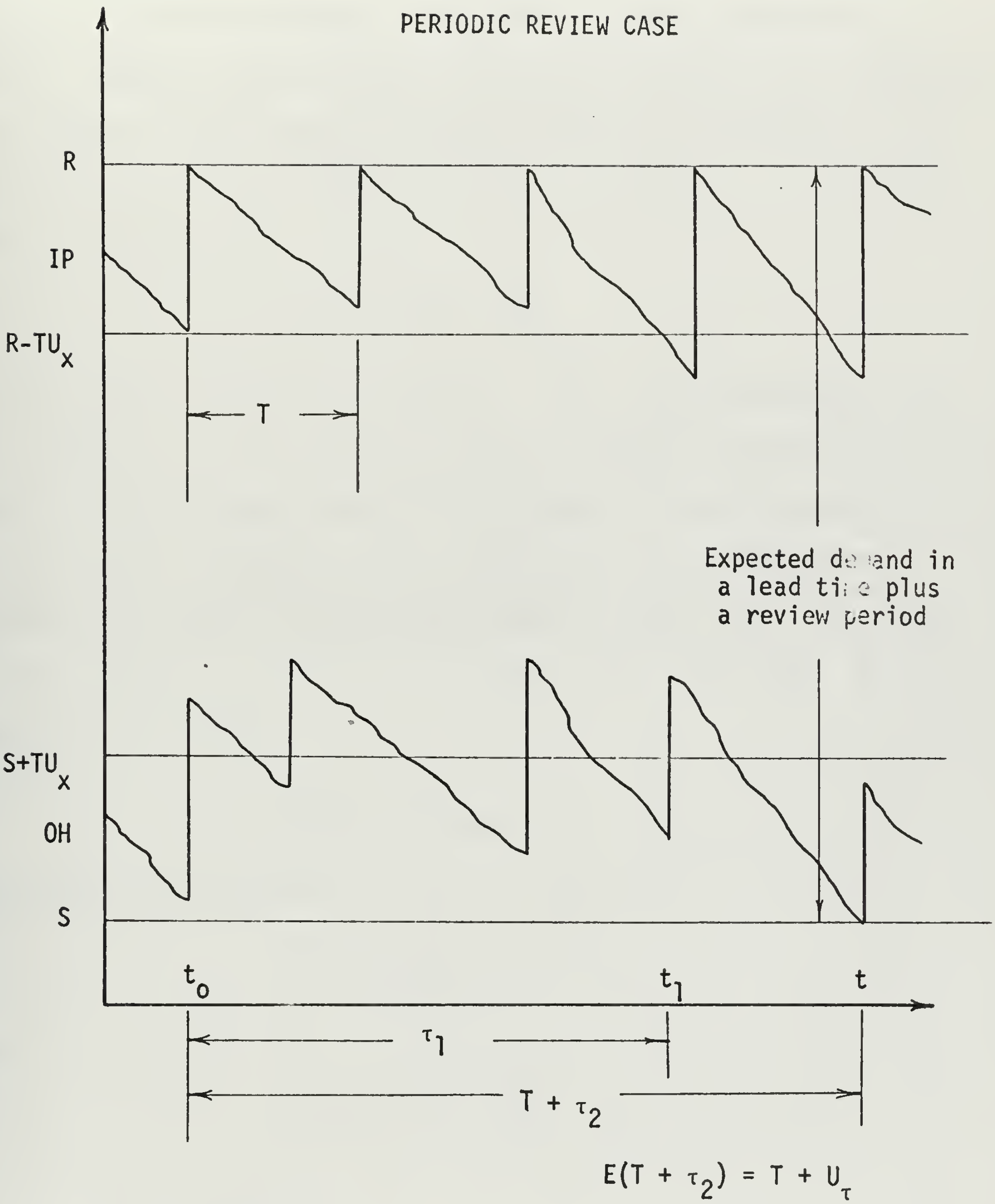
$$\text{MINIMIZE} \qquad R - \frac{TU_X}{2} - U$$

$$\text{SUBJECT TO:} \qquad \int_R^\infty h(x,\tau+T)\,dx \le e$$

$$T \ge T'$$

It is conceivable that there is a further constraint of the form,

$$A \le T\,U_X \le B$$



PERIODIC REVIEW CASE
R
IP
R-TU_x
T
Expected demand in
a lead time plus
a review period
S+TU_x
OH
S
t_o
t_1
t
τ_1
T + τ_2
$E(T + \tau_2) = T + U_\tau$

Figure II.1

where A is the minimum practicable order size and B is the maximum amount that can be shipped at one time.

Since T is independent of R, the objective function, call it L(R,T), is clearly increasing in R so that the first constraint is active. However, R is an implicit function of T by the first constraint, and it must be shown that the objective function is increasing in T in order to claim that the second constraint, involving T, is also active. Now, by the chain rule,

$$dL/dT = dR/dT - \frac{U_x}{2}$$

so that $dL/dT > 0$ implies that $dR/dT > \frac{U_x}{2}$. If, in accordance with our argument, e is less than 0.50, which is certainly desirable, then R must at least be greater than the mean demand in a lead time plus a review period. Therefore if T is increased by one day then R must increase by at least the mean daily demand, or $dR/dT > U_x$. Thus, the second constraint is active and the optimal order up to level, R^* is that R which satisfies

$$\int_{R^*}^{\infty} h(x,\tau+T^*)dX = e.$$

The optimal review period, T^*, is

$$T^* = \text{Max}\left(\frac{A}{U_x}, T'\right)$$

and

$$E(OH) = R^* - \frac{T^* U_x}{2} - U.$$

2. <u>Continuous Review</u>

The model for the continuous review case can be stated as;

$$\text{MINIMIZE} \quad \frac{Q}{2} + R - U$$

$$\text{SUBJECT TO:} \quad \int_R^\infty h(x,\tau)\,dx \le e$$

$$Q/U_x \ge T'$$

where

$$E(OH) = \frac{Q}{2} + R - U$$

and the expected time between orders, T, is

$$\frac{Q}{U_x} = T.$$

R^* is determined by

$$\int_{R^*}^\infty h(x,\tau)\,dx = e$$

and the optimal reorder quantity, Q*, is that which satisfies

$$Q^* = \text{Max}(A,\ T'\ U_x).$$

E. THE DISTRIBUTION OF LEAD TIME DEMAND

There is, no doubt, a certain lack of probabilistic sophistication in these models, but the overriding concern of this paper was to provide a model that produces practical results based on an understandable and direct approach, simplified where necessary so that the method could be tested by simulation, and if shown to be feasible, adopted as practice by managers of ammunition inventory systems.

Nonetheless, we are not yet at this point with the model. So far, the results have been presented in terms of the symbolic probability density function $h(x,\tau+t)$. Clearly, the problem cannot be solved unless an analytical expression for $h(x,\tau+t)$ can be found. This expression was implicitly assumed to exist. Unfortunately, there are only certain special cases where the necessary convolution is tractable. The most

promising of these special cases is when f(x,t) is the Poisson density function with parameter tU_x, $g(\tau)$ is distributed gamma and the resultant lead time demand distribution is negative binomial. See Taylor [9].

However, the data presented in Ref. 8, while intuitively permitting the hypothesis that lead time has the gamma distribution, does not permit the assumption that daily demand is Poisson. One would expect such a result in the case of ammunition since demands do not occur one unit at a time. In addition, demands are in general high and not the rare events that the Poisson density describes.

Furthermore, none of the daily demand data for the 12 ammunitions of Ref. 8 can pass, at a significance level of 0.10, the Kolmogorov-Smirnov test of the hypothesis that the demand distribution is normal with mean and variance estimated from the data, as given in Lilliefors [5]. There does not appear to be any consistent distribution apparent from the daily demand histograms presented in the III MAF Study. Perhaps daily demand could be fitted to a truncated normal but this does not lead anywhere because the n-fold convolution of a truncated normal random variable is not something which comes immediately to mind. Certain of the histograms for low demand items appear to have an exponential form, but again, while the n-fold convolution of the exponential distribution is gamma, the integration required for two gamma distributions with different parameters does not appear to have an analytical solution.

To be brief, real world data is not amenable to analytical derivation of the probability density function of lead time demand and so another approach must be taken.

F. A NORMAL APPROXIMATION

It has already been assumed that daily demands, x_i, are independent random variables drawn from the same distribution and that the distribution of the lead time, in reality a discrete random variable, is independent of daily demand. Thus, the demand in a lead time plus a review, call it Y, is the sum of independent random variables,

$$Y = \sum_{i=1}^{\tau+T} X_i .$$

Considering the conditional distribution of Y given τ, we have that, since T is fixed by the second constraint,

$$E(Y/\tau) = (\tau + T)U_X$$

and

$$V(Y/\tau) = (\tau + T)\, V_X .$$

It can be shown that

$$E(Y) = U = (U_\tau + T)\, U_X$$

$$V(Y) = V = (U_\tau + T)\, V_X + U_X^2\, V_\tau$$

so that Y has some distribution, h(y), with the above mean and variance. As Y is the sum of independent, identically distributed random variables, we have from the central limit theorem that, as the lead time gets large, $f(y/\tau)$ approaches the normal distribution with mean $(\tau+T)U_X$ and variance $(\tau+T)V_X$. Then, since

$$h(y) = \int_{-\infty}^{\infty} f(y/\tau)g(\tau)d\tau$$

one can conjecture that Y, lead time demand, is distributed approximately N(U,V). This approximation is suggested in Tully [6] and Hadley [3] and discussed in Clark [2].

If it is assumed that this is the case, then the solution for the optimal R is given by

$$\overline{F}\left(\frac{R^*-U}{V^{1/2}}\right) = e$$

where the left hand side represents the right tail of the standard normal distribution at R*. The value of the standard normal variate associated with 1-e, Z_{1-e}, can be used to solve for R* as follows:

$$R^* = Z_{1-e}((U_\tau+T)V_X + U_X^2V_\tau)^{1/2} + U_X(U_\tau+T). \qquad \text{(II.1)}$$

Using this approximation, it is a simple matter to construct a computer program that reads in current data samples of lead times and daily demands, computes the appropriate means and variances and calculates R*, given T*, for designated values of e. Similarly, in the continuous review case, where one is interested in the demand in a lead time,

$$E(Y/\tau) = \tau E(X)$$
$$V(Y/\tau) = \tau V(X)$$

and

$$E(Y) = U_\tau\ U_X$$
$$V(Y) = U_\tau V_X + U_X^2\ V_\tau$$

so that R* can be approximated by

$$R^* = Z_{1-e}\ (U_\tau\ V_X + U_X^2\ V_\tau)^{1/2} + U_\tau\ U_X\ . \qquad \text{(II.2)}$$

III. EXAMPLES

A. DATA

In order to test the validity of the model two cases were taken from data presented in Ref. 8; one representing low demand, specifically 4.2 in. illumination, and the other representing high demand, 105 mm. HE. In both cases the periodic review model was used and the constrained value of T* was taken as 7 days in accordance with the results of [8].

The lead time data from the time period January to April 1969 resulted in a mean, U_τ, of 48.05 days, and variance, V_τ, equal to 506.69. (A sample value of 146 days was discarded as being an extreme outlier.) The lead time data was tested for its fit to the normal distribution using the results of Ref. 5. The hypothesis that the data came from a normal distribution could not be rejected at the 0.10 significance level using a Kolmogorov Smirnov test of goodness of fit.

The data was then separated into two categories, pyrotechnic and high explosive, and no significant difference in the mean and variance of the two resulting samples was found using the t-test for two independent samples and the F-test respectively. The data was also separated according to whether the requisition was filled in Sasebo or not, since Sasebo was the primary source of supply in this case. Curiously enough, the non-Sasebo sample mean lead time was slightly less than the sample mean of the Sasebo data in spite of the fact that some non-Sasebo requisitions were filled in the United States.

In summary, there appeared to be no statistical reason to assume that the lead time distribution was different for each ammunition type and thus the assumption was made for the computations that each ammunition type faces the same lead time distribution, with a mean of 48.05 days and

a variance of 506.69. An important feature of this model is that it does not matter what the lead time distribution is under the normal approximation used in the calculations.

It has already been mentioned that daily demand data for all 12 categories of ammunition contained in Ref. 8 did not fit a normal distribution and that there is no underlying distribution immediately evident from the data.

B. MODEL RESULTS

1. The Low Demand Case

The probability that daily demand is zero as estimated from the data is greater than 0.50. The results of the calculations for this case were U_x, mean daily demand, equals 113.16, and V_x, variance, equals 62,874.30.

(In this case and the next there is no explanation apparent for the significant discrepancy between these values and those given in Ref. 8.)

Equation II.1 gives the following results (rounded to the nearest integer) for various levels of protection, (1-e);

LEVEL OF PROTECTION	R*	EQUIVALENT IN DAYS OF AMMUNITION
0.999	15976	141
0.995	14355	127
0.990	13566	120
0.980	12708	112
0.970	12162	107
0.960	11752	104
0.950	11418	101
0.940	11118	98
0.925	10771	95
0.900	10267	91

2. The High Demand Case

The probability that daily demand is zero as estimated from the data is less than 0.05. The results of the calculations for this case are as follows; U_x, mean daily demand, equals 5628.54 and V_x, variance, equals 22676030.0.

LEVEL OF PROTECTION	R*	EQUIVALENT IN DAYS OF AMMUNITION
0.999	716256	127
0.995	648649	115
0.990	615766	109
0.980	579990	103
0.970	557235	99
0.960	540136	96
0.950	526194	93
0.940	513698	91
0.925	499230	89
0.900	478185	85

C. SIMULATION OF THE SYSTEM

1. Methodology

To test the preceding results, a straightforward simulation of a standard inventory situation was programmed in GPSS, a computer language specifically designed for simulations of this type. In this computer model, demands are generated daily, the size of the demand determined by using the language's built-in Monte Carlo technique against the empirical daily demand distribution resulting from data given in the III MAF Study. No assumptions were made about the daily demand distribution other than it is continuous.

These demands were presented to a storage facility which met them if possible and, if not, backordered the balance. The inventory position was reviewed every 7 days in computer time and the amount to be ordered at each review was computed from the formula, Q = R*-(OH+OO-BO). This order was placed and a lead time assigned to it using the empirical

distribution of lead times which was assumed to be continuous. It should be noted that in GPSS all values are truncated to integers, resulting in lead times of whole days and demands of whole rounds. Orders were delayed in the system according to their assigned lead times and were <u>permitted to cross</u>. When an order arrived at the dump it first filled outstanding backorders and the balance was placed in the dump (on hand) inventory.

The system was initialized with the expected values of rounds on hand and on order as computed by the model and permitted to run for 50 days before statistics were kept in order to stabilize the system after the initial start up. Following the 50 day stabilization period, the model was run for 10,000 days under the same input parameters to provide statistically sound estimates of its operating characteristics.

Further, for each R*, a sample of nine two-year periods was taken, changing the random number seeds before each single two-year period to provide a basis for historical records of system operation (an example of which is presented in Fig. III.1), to provide another estimate of system characteristics and to vary the random number sequence to test the sensitivity of the program to the random numbers generated.

P(out) has been defined as the limit as n approaches infinity of n'/n where n' is the number of days on which backorders occur and n is the number of days of operations of the system. Therefore, the simulation estimate of P(out) is

$$\overline{P}(\text{out}) = \frac{\text{the number of days on which a backorder occurs}}{\text{10,000 days of operation}}.$$

SAMPLE TIME HISTORY OF OPERATIONS
HIGH DEMAND CASE

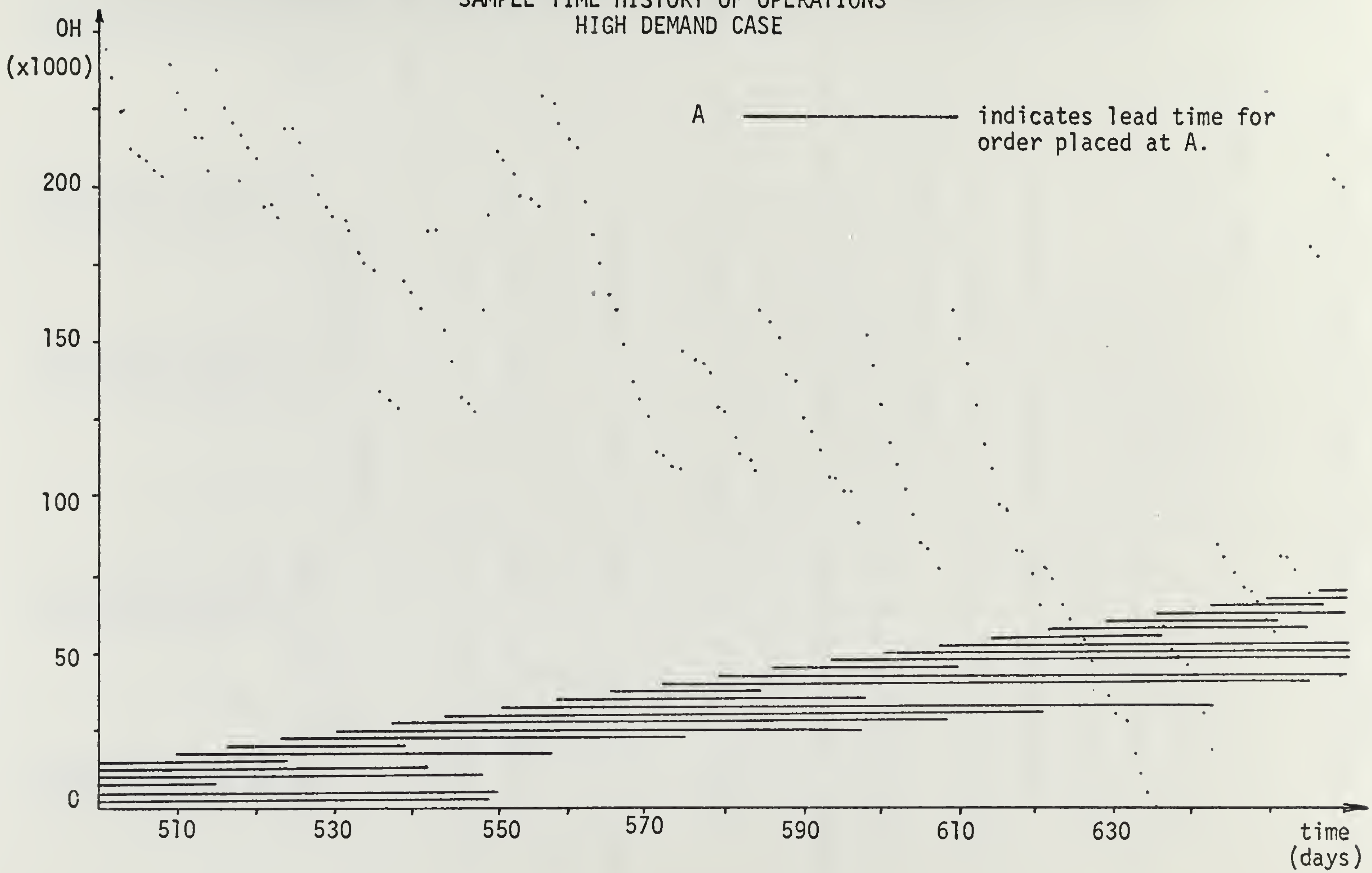


Figure III.1

GPSS automatically keeps track of the necessary data to compute average on-hand inventory and this figure is given in the standard GPSS output.

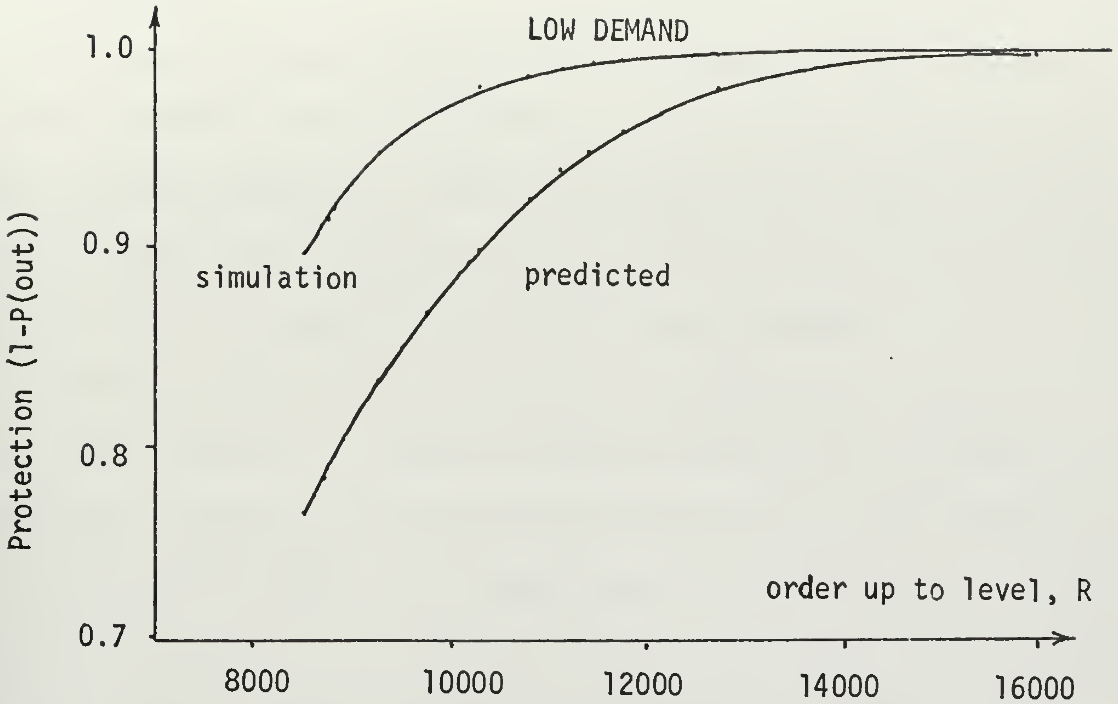
2. Simulation Results

The most immediate and striking result of the simulation was that the protection offered by the decision rules was in excess of that predicted by the model. Although Ref. 6 corroborates the conservatism of the normal approximation in an ammunition study, the discrepancy was quite large in the examples. Excess protection in the high demand case was greater than in the low demand case.

In spite of the discrepancy between predicted protection levels and the levels realized in the simulation, the predicted average on-hand inventories are quite close to those realized in the simulations. In the high demand case average on-hand inventories are within 0.04% of the predicted values and within 5.5% in the low demand case.

The predicted and actual protection levels (1-P(out)) are shown versus the order up to levels, R*, in Figs. III.2 and III.3 for the low demand and high demand cases respectively. Results for the predicted and actual average inventory for various levels of predicted protection are presented below.

	AVERAGE ON HAND INVENTORY			
PREDICTED	HIGH DEMAND		LOW DEMAND	
PROTECTION	PREDICTED	ACTUAL	PREDICTED	ACTUAL
0.999	426104	425980	10143	10355
0.995	358494	358373	8522	8734
0.990	325607	325490	7733	7946
0.980	289822	289714	6875	7088
0.970	267056	266959	6328	6542
0.960	249945	249860	5918	6133
0.950	235993	235918	5584	5801
0.940	223485	223422	5284	5503
0.925	208999	208954	4936	5160
0.900	187921	187922	4431	4664



COMPARISONS OF PREDICTED PROTECTION VS. SIMULATION RESULTS

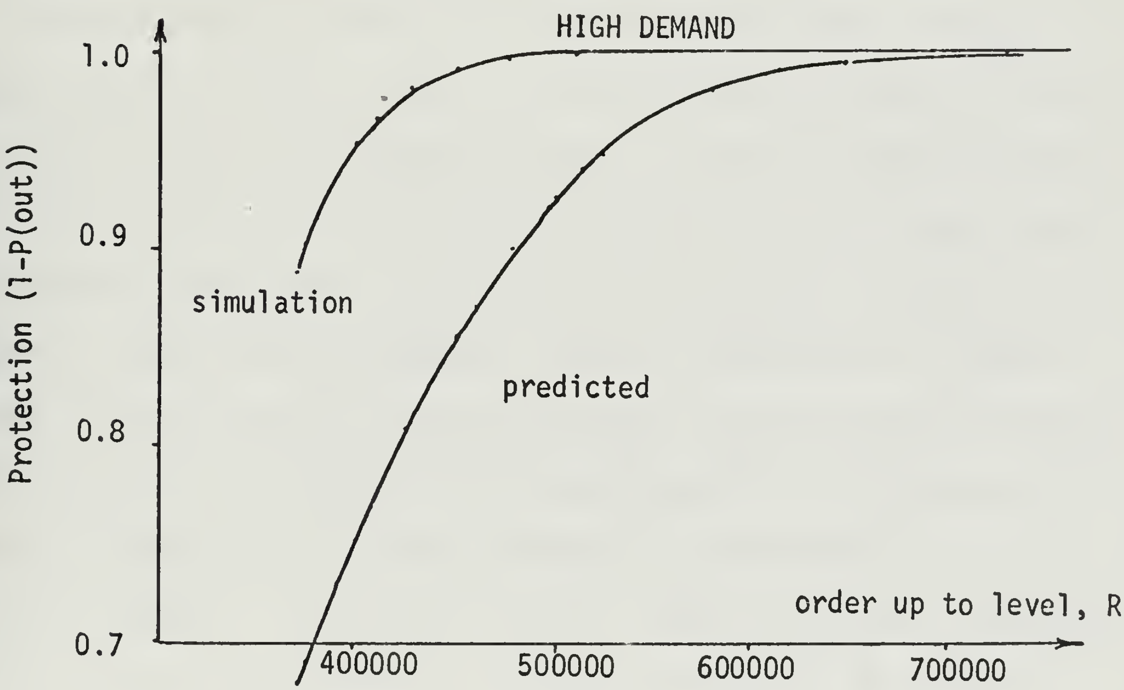


Figure III.3

D. DISCUSSION

The accuracy of the predictions for the expected on-hand inventory is not surprising since that prediction is a function of mean lead time demand and mean demand in a review period, parameters which are derived from the means and variances of the lead time and daily demand data and not from any consideration of the distributions involved.

However, in the case of the protection offered by an order up to level, the prediction is completely dependent upon the distribution used. P(out) for a given R is in fact the right hand tail of the distribution of lead time demand at R. The implication of the simulations is clear; the distribution of lead time demand is not normal and the normal approximation results in large discrepancies in the tails. However, in the examples presented, the error is on the side of conservatism, i.e., increased protection, and thus the results are still of practical value.

The discrepancy in protection provided by the model, although anticipated by Refs. 2, 4, and 6, was large enough to cause the author to check the simulation program in detail. Analysis of the lead time and daily demand distributions to check the effect of the random number generators revealed that the simulation distributions followed the empirical distributions extremely closely. Nonetheless, the sample means and variances used in the calculations do not correspond exactly to the values realized from the continuous empirical distribution in the simulation since the sample parameters are estimates of the distribution parameters. Accordingly, certain reorder levels were recomputed using the means and variances of the simulation lead time and daily demand distributions. The results were still quite conservative so that this factor does not explain the difference between predicted and actual performance.

The simulation program itself was thoroughly checked and since it was rather simple it is felt that the source of the error is not there. The only conclusion is that the discrepancies are caused by permitting orders to cross when the model makes the opposite assumption and/or the skewness of the lead time demand as discussed in Clark [2]. The latter cause is intuitively the more likely.

IV. DISCUSSION

A. STOCHASTIC LEAD TIMES

A survey of work in the application of inventory theory to systems that face stochastic lead times is found in Bramson [1]. While the literature presents no method to analytically handle the case of orders crossing, one alternative suggested in Ref. 1 is to redefine lead time as the time interval between the nth order and the nth receipt and proceed with an analysis such as the one in the present paper. Obtaining a lead time data sample on the basis of that definition requires a historical record of operations but, given that, does not appear to present any particular practical difficulties. Unfortunately, the author did not have access to such a record for the example. It is possible to generate an historical record of the system operation from the simulation and obtain a sample of lead times as defined above. However, it was felt that since a simulated sample is twice removed from reality it would be difficult to make any practical claims for the results.

Another difficulty in modelling stochastic lead times with large variances is the intuitive feeling that orders do not cross in actual operations although consecutive orders may arrive together. Hadley and Whitin [4] support this conjecture. Orders arriving in sequence imply that the independence assumption for lead times with large variances does not hold and there must be some dependence or serial correlation between orders. Hadley and Whitin [4] state that one immediate effect of the independence assumption in this case is underestimation of the variance of lead time demand and, therefore, dangerously low order up to levels.

Further, if orders do not in fact cross as determined from observation, then considerable effort must go into analyzing the requisitioning system in order to generate a lead time distribution which has the necessary correlation. In the ammunition case such an effort would rapidly lead to analyzing factors as diverse as shipping schedules and production rates. Answers derived from computer simulations loom larger in the face of the necessity to undertake the analysis of all the interlocking parts of the system and the resultant intractable analytical complexities. The goal constraint model is <u>clearly the easiest to simulate</u> because it is essentially driven by only one consideration -- raising R to a level that provides satisfactory protection for a given T.

B. SKEWNESS OF LEAD TIME DEMAND

Skewness is measured by the third central moment,

$$U_3 = \int_{-\infty}^{\infty} (X-U)^3 f(x)\, dx.$$

Clark [2], presents a practical but involved method to account for the fact that the normal approximation to lead time demand is weakest in the tails of the distribution. Unfortunately, this is precisely the part of the distribution that the model is interested in. If U_{3X} is the third central moment of the daily demand distribution and $U_{3\tau}$ the analogous moment for lead times, then the third central moment of lead time demand, U_3, is given by

$$U_3 = U_{3X}U_\tau + 3U_X V_X V_\tau + U_X^3 U_{3\tau} \qquad \text{(IV.1)}$$

However, the third central moment of any symmetric distribution, such as the normal, is zero. Therefore, Eq. IV.1 shows that even if the

daily demand and lead time distributions are normal the distribution of lead time demand is not.

Clark [2] contends that if U_3 is positive then the right hand tail of the normal approximation to the lead time demand density function is below the corresponding tail of the actual lead time distribution and thus predicted protection is too optimistic. However, two ammunition studies, Ref. 6 and this paper, deal with empirical data where predicted protection is conservative in the portion of the right hand tail of interest. The third central moment as given by Eq. IV.1 was estimated for both examples of this paper and was positive in both cases. Nonetheless, as stated above, the predicted protection levels were conservative. A potential explanation is the effect of orders crossing. However, in the light of Clark's observations, a simulation to test predicted protection seems called for in future situations.

C. COMPARISON OF EXAMPLE RESULTS

It would not be accurate to directly compare the goal-constrained simulation results of this paper to the minimum cost simulation results of Ref. 8 because, as has been indicated, there is a significant difference in the means and variances of the daily demand samples used in both studies. Accordingly, in order to make a more valid comparison, order up to levels were computed from Eq. II.1 using the means and variances given in Ref. 8 and the results were compared to the minimum cost figures derived by that study. However, investigation of the method used by Ref. 8 showed that the optimal minimum cost order up to level, which was derived by simulation, was constrained by a requirement to have no stockouts in the simulation period of operations. The

net result of that simulation was to minimize the cost of holding on hand inventory subject to constraints that there be no stockouts in the simulation and the review cycle be 7 days or greater. Thus, both studies simulated essentially the same thing in the same way.

The most pertinent outcome of this comparison then is this; both studies resorted to a computer simulation in the final analysis to derive their results and, as was argued in Chapter I, the natural way to simulate the situation was to minimize expected on hand inventory subject to a protection constraint and a constraint on order frequency.

V. CONCLUSIONS

This study effort was undertaken to investigate a promising alternative to the standard economic approach for deriving inventory policy. It was hoped that this alternative, categorized as the goal constraint approach, together with a simple analytical model could be presented as a practical way to solve the inventory control problem in certain military situations.

The paper derived such a goal constraint model in the context of an ammunition distribution system based on two principal assumptions:

1. That decision rules resulting from an assumption that orders do not cross are valid approximations for the case where they can cross.

2. That the distribution of lead time demand is approximately normal.

These assumptions were necessary to approximate a situation where certain intractable problems did not permit an analytical solution. These problems arose from the effect of stochastic lead times and the determination of lead time demand. Because of the approximations, the analytical model was tested by computer simulation to determine if its results were valid. The simulation showed that the protection levels predicted by the model, although on the conservative side, were inaccurate for the case where orders were permitted to cross. Thus, while the goal constraint approach retains its appeal, it is difficult to maintain confidence in the specific model given.

In addition, if one considers application of the model to other situations, preceding discussion has shown that several important

complexities can occur in practice. For example, a problem arises if observation of actual operations indicates that orders do not cross and yet the empirical lead time distribution permits the possibility that they will cross under an independence assumption. Reference 4 contends that the underestimation of the variance of the lead time distribution that will result from the independence assumption will be reflected in reorder levels that are too low. In order to properly account for the correlation of the lead times, it would be necessary to conduct extensive study of the requisitioning system. Since there is small hope for analytical results, such an effort probably will resort to computer simulation techniques.

On the other hand, it is essential that skewness of the lead time demand distribution be checked when using the model. The most direct check would be to test decision rules in a simple simulation of the inventory system. A more complex alternative is to generate a lead time demand distribution by computer techniques using the empirical lead time and daily demand distributions as input. In this way the right hand tail of the lead time demand distribution can be compared directly to the normal approximation in tabular or graphical form. Of course, such an effort is not necessary if the lead time demand distribution can be constructed from historical records.

It has become obvious by now that the discussion turns inexorably to computer simulation as the only method which can provide the necessary confidence in derived inventory policies in the face of the kind of uncertainties and approximations that have arisen in the course of this analysis. Therefore, the final conclusions are these:

1. The most practical approach to derive decision rules is by a computer simulation formulated on the basis of the goal constraint concept. This method is a simple, direct and useful way to generate inventory policy.

2. Order up to levels based on the normal approximation provide a place to start in the simulation process.

3. The simplicity of the goal constraint concept permits the bulk of study effort to be applied to data analysis and forecasting considerations, in themselves difficult problems.

Therefore, it is recommended as a practical matter that personnel responsible for the operation of inventory systems as described herein consider adapting the goal constraint approach to computer simulation techniques in order to derive inventory policy in a straightforward manner.

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13. ABSTRACT

The paper contends that the standard economic approach to inventory control may not be as valid in a military context as a goal constrained model which optimizes a performance criteria. The argument is applied to an ammunition system in a combat zone and a model is formulated to minimize expected on-hand inventory subject to constraints on required protection and order size or frequency. The model assumes that orders do not cross. Two examples, using Marine Corps data, were tested by a computer simulation that permitted orders to cross. The simulation showed that predicted average on hand levels were accurate but predicted protection was conservative. It was concluded that the model is best solved by simulation in view of the uncertainties caused by stochastic lead times and the skewness of lead time demand.

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